Estimating Radial Velocity of Fixed Beds with Low Tube-to-Particle Diameter Ratios

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The Ergun equation based on the effective spherical diameter is universally valid for various shapes of granular packings. It, however, was derived from the assumption of infinite tube-to-particle diameter ratio without considering the wall effect. Although some improvements were made by Mehta and Hawley (1969) to correct this, the application of the Ergun equation is still restricted to cylindrical columns with a packing porosity of less than 0.5. To modify the Ergun equation to noncylindrical flow space with or without a wall, it was substituted into the empty tube pressure drop equation by introducing an effective tube diameter so that the pressure drop can be predicted just from the free flow space and the wetted area involved. This treatment offers the basis for a new method in velocity distribution prediction.

Introduction

It is reasonable to consider that there is a certain velocity profile corresponding to the global characteristic of the packing. Obviously, the measurement with a hot-wire anemometer cannot provide this representative velocity profile since it must be placed at the exit of a packed bed. The recently developed optical method appears to be a powerful tool in obtaining the characteristic velocity. Stephenson and Stewart (1986) used a marker tracing method to observe the fluid motion with a video camera in a transparent packed bed. It was found that the local superficial velocity attained its global maximum and minimum at distances near 0.2 d_p and 0.5 d_p from the wall, which was not observed by the hot-wire anemometer. A similar observation was also made by McGreavy et al. (1986), who used a Doppler-laser anemometer to study velocity profiles with the maximum velocity being approximately 0.3 d_p from the wall.

Theoretical prediction of velocity distribution in a fixed bed has been based on the Brinkman equation or the Ergun equation. The Brinkman (1947) equation is derived from the assumption that the packed bed can be modeled as a fluid continuum, thereby allowing the packing resistance term to be added to the Navier-Stokes equation. The prediction made by Johnson and Kapner (1990) showed the peak velocity near the wall to be almost 3.5 times the average value, which is much higher than experimental findings. This situation calls for other solutions to the problem under consideration with the Ergun equation method being an alternative one. The application of the Ergun equation is based on the bundle-tube model, which means that the flow through the packed bed is somewhat analogous to the flow through a bundle of tubes (Schwartz and Smith, 1953). This model has been improved by Cohen and Metzner (1981) who divided the bed into three regions: the wall region which extends from the wall over a distance of one pellet diameter, the intermediate region with a width of five pellet diameters, and the central region. Each region has a uniform, but stepwise, effective diameter. Consequently, only the cross-sectional average velocity of the three regions can be obtained from this model. In order to obtain the point velocity, Schnitzlein (1993) considered the bed composing of a bundle of parallel tubes characterized by hydraulic diameters. The hydraulic diameter is a function of porosity since it is related to the flow space and wetted area; therefore, the tubes will have different hydraulic diameters leading to different fluid velocities as all the tubes possess the same pressure drop as the packed bed. However, this model failed to provide a reasonable prediction near the wall, and it was estimated that this might originate from neglecting the viscous effects at the container wall. We also believe the failure took place because the porosity near the wall is larger than 0.5, which is beyond the validity of the Ergun equation. Ziolkowska and Ziolkowski (1993) introduced a local effective viscosity to relate the gas kinetic energy dissipation due

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to the friction on the surface of the pellets, the tube wall, and the shear stresses caused by fluid motion and the radial dispersion in the gas stream. The effective viscosity was an empirical parameter which was based on the measurements of the radial profile of the superficial gas velocity just above a cross section at the exit of the bed. Previous studies demonstrate that some further research is quite necessary for the fluid motion prediction. In this article, a new physical model is established which enables us to predict the distribution of superficial velocity exactly from the wall ($r = R$) to the center ($r = 0$) without introducing any empirical parameters as encountered in those of Schnitzlein (1993) and Ziolkowska and Ziolkowski (1993).

Theoretical Development

The pressure drop for fluid flowing through an empty cylindrical tube under isothermal condition is described by

$$\Delta P = 4f\frac{L}{d_t}\frac{\rho u^2}{2} \tag{1}$$

where f is the friction factor for an empty tube, L is the length of a packed bed, d_t is the packed bed diameter, ρ is the density of the fluid, and u is linear velocity. When this equation is applied to a fixed bed, Eq. 1 can be modified as

$$\Delta P = 4f_p\frac{L}{d_e}\frac{\rho \hat{u}^2}{2} \tag{2}$$

where

$$\hat{u} = \bar{u}/\epsilon, \quad \text{and} \quad d_e = \frac{2}{3}\frac{\epsilon}{1-\epsilon}d_p \tag{3}$$

$\bar{u}$ is superficial velocity. Substituting the expressions in Eq. 3 into Eq. 2, it will obtain

$$\Delta P = 3f_b\frac{L}{d_p}\frac{1-\epsilon}{\epsilon^3}\rho\bar{u}^3 \tag{4}$$

The Ergun (1952) equation is expressed as

$$\Delta P = F_b\frac{L}{d_p}\frac{1-\epsilon}{\epsilon^3}\rho\bar{u}^2 \tag{5}$$

The following relationship between f_b and F_b is obvious (f_b is the friction factor for a packed bed, and F_b is the friction factor in the Ergun equation)

$$f_b = \frac{F_b}{3} \tag{6}$$

Therefore, in terms of interstitial velocity $\hat{u}$, Eq. 2 becomes

$$\Delta P = \frac{4}{3}F_b\frac{L}{d_e}\frac{\rho\hat{u}^2}{2} \tag{7}$$

where

$$F_b = \frac{150}{R_e} + 1.75, \tag{8}$$

and

$$R_e = \frac{d_p\bar{u}\rho}{\mu(1-\epsilon)} = \frac{3}{2}\frac{d_e\hat{u}\rho}{\mu} \tag{9}$$

R_e is the particle Reynolds number, $d_p u\rho/\mu$. This validity of Eq. 7 is verified in the prediction of pressure drop with wall effect (Mehta and Hawley, 1969), and therefore becomes the basis of a new velocity profile predicting method.

Velocity Distribution Predicting Method

Near the wall of the packed-bed reactor, flow channels are larger and offer less resistance, at the same time, wall skin friction tends to reduce the flow, so these two factors operate against each other and result in complex fluid dynamics. Therefore, the role of the wall should be properly accounted for. The wall effect in this article is considered to extend to all radial positions in the bed, and the flow space confined between the wall ($r = R$) and a certain cylindrical surface $r_i(0 \le r_i \le R)$ is under investigation (r is the radial position, and R is the radius of a tube).

The number (n) of pellets contained in a differential element $(2\pi rdr)L$ is

$$dn = \frac{[1-\epsilon(r)](2\pi rdr)L}{\pi d_p^3/6} \tag{10}$$

The external surface area of the pellets within the circular region between r_i and R will be (d_p is the packing diameter)

$$A_p = \int_{r_i}^{R} dn\cdot\pi d_p^2 = 12\frac{L}{d_p}\int_{r_i}^{R}\pi r[1-\epsilon(r)]dr \tag{11}$$

From the pellet area A_p and the wall surface area A_w

$$A_w = 2\pi RL \tag{12}$$

With the flow volume in this region

$$V = L\int_{r_i}^{R}2\pi r\epsilon(r)dr \tag{13}$$

the hydraulic radius can be obtained

$$R_h = \frac{V}{A_w + A_p} = \frac{L\int_{r_i}^{R}2\pi r\epsilon(r)dr}{2\pi RL + 12\frac{L}{d_p}\int_{r_i}^{R}\pi r[1-\epsilon(r)]dr} \tag{14}$$

and thus the effective diameter is obtained

$$d_e = 4R_h = \frac{4\int_{r_i}^{R} r\epsilon(r)dr}{R + \frac{6}{d_p}\int_{r_i}^{R} r[1-\epsilon(r)]dr} \tag{15}$$

Substituting the expression for $\epsilon(r)$ into Eq. 15 then yields d_e (effective tube diameter).

Denoting the effective diameter of the region confined between R and $r_i + \Delta r$ as d_{e1} and that between R and $r_i - \Delta r$ as d_{e2}, the pressure drops of fluid flowing through these two regions will be

$$\Delta P_1 = \frac{4}{3} F_b \frac{L}{d_{e1}} \frac{\rho \hat{u}_1^2}{2} \tag{16}$$

and

$$\Delta P_2 = \frac{4}{3} F_b \frac{L}{d_{e2}} \frac{\rho \hat{u}_2^2}{2} \tag{17}$$

For such parallel flows, the pressure drops are identical

$$\Delta P_1 = \Delta P_2 = \Delta P \tag{18}$$

where ΔP is the pressure drop of the entire bed.

After the bed pressure drop and effective diameters are known, the interstitial velocities in these two regions can be obtained from Eqs. 16 and 17

$$\hat{u}_1 = \sqrt{\frac{3}{2}\frac{\Delta P}{F_b \rho L} d_{e1}}, \tag{19}$$

and

$$\hat{u}_2 = \sqrt{\frac{3}{2}\frac{\Delta P}{F_b \rho L} d_{e2}} \tag{20}$$

The volumetric flow rates V_1 in the region $(R, r_i + \Delta r)$ and V_2 in the region $(R, r_i - \Delta r)$ will be obtained according to the following equations

$$V_1 = \bar{\epsilon}_1 A_1 \hat{u}_1 = \bar{\epsilon}_1 \pi \left[R^2 - (r_i + \Delta r)^2\right]\hat{u}_1 \tag{21}$$

$$V_2 = \bar{\epsilon}_2 A_2 \hat{u}_2 = \bar{\epsilon}_2 \pi \left[R^2 - (r_i - \Delta r)^2\right]\hat{u}_2 \tag{22}$$

where $\bar{\epsilon}$ is the average porosity over the cross-sectional area between R and r_i, and it is used here to account for the actual flow area. (A_1 is the flow area of the ring $(R, r_i + \Delta r)$; A_2 is the flow area of the ring $(R, r_i - \Delta r)$.) It is obtained from

$$\bar{\epsilon} = \frac{1}{R^2 - r_i^2}\int_{r_i}^{R} 2r\epsilon(r)dr \tag{23}$$

V_2 is always greater than V_1 since a larger flow area is occupied, and this increment is the result of the increased flow area $(A_2 - A_1)$. Therefore, the superficial velocity $\bar{u}(r)$ over the circular band interval $(r_i + \Delta r,\ r_i - \Delta r)$ can finally be obtained

$$\bar{u}(r) = \frac{V_2 - V_1}{A_2 - A_1} \tag{24}$$

When the width of the circular band $(r_i + \Delta r,\ r_i - \Delta r)$ is small enough, that is, $A_2 - A_1$, the velocity in Eq. 24 will approach its real local point value (r is the radial differential distance).

Results and Discussion

The prediction of velocity distribution is first compared with the optimal measurement by Stephenson and Stewart (1986). The porosity distribution for their experimental system (d_t = 75.5 mm and d_p = 7.035 mm) is predicted according to Mueller (1991) and shown in Figure 1. The predicted velocity profile is shown in Figure 2 at R_{ep} = 236.9 with $\Delta r = 0.025\ R$

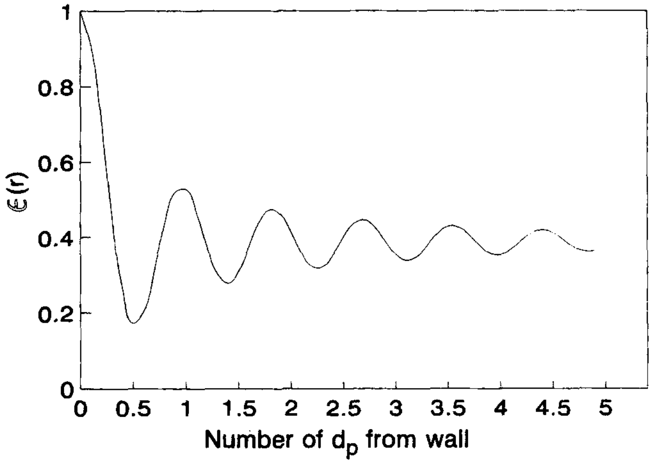

Figure 1. Porosity distribution (according to Mueller, 1991).

d_t = 75.5 mm; d_p = 7.035 mm.

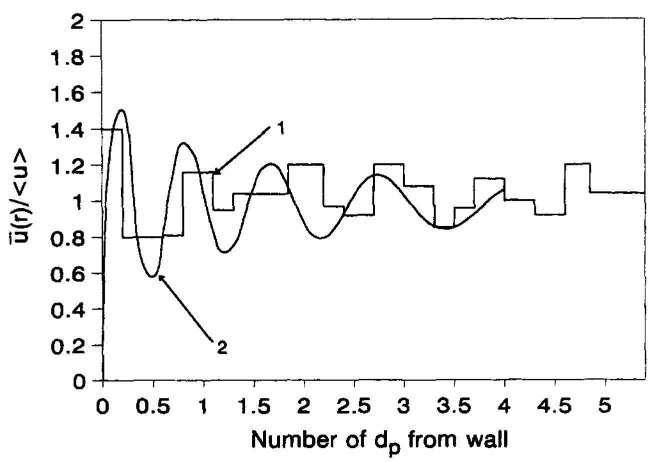

Figure 2. Velocity distribution: measurement and prediction.

(1) Optical measurement of Stephenson and Stewart (1986); (2) method developed in this article.

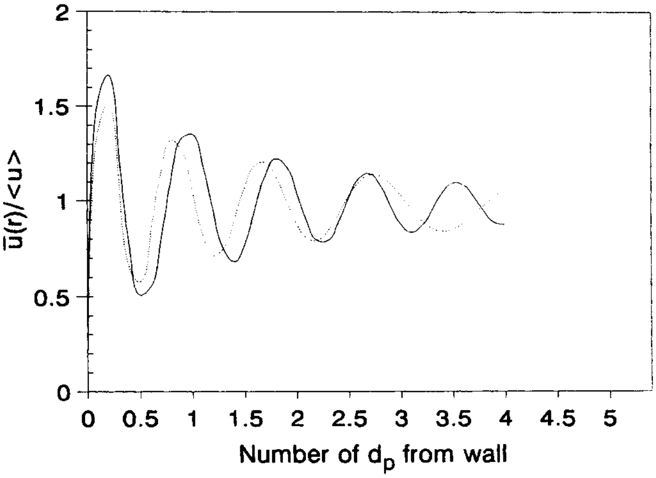

Figure 3. Influence of width of circular band on precision of velocity profile.

—— $\Delta r = 0.01\ d_p$; ···· $\Delta r = 0.025\ R = 0.268\ d_p$.

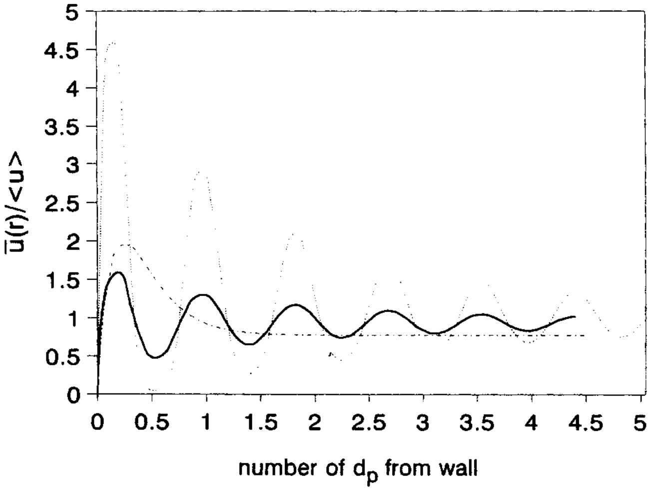

Figure 4. Predictions of velocity distribution by different methods.

—— Ergun equation (this article); ···· Brinkman equation (Johnson and Kapner, 1990); -·-·-·- Brinkman equation (Tsotsas and Schlünder, 1988).

according to Stephenson and Stewart's experiment in order to make the comparison under identical conditions. The prediction shows that the largest value of the respective superficial velocity $\bar{u}(r)/\langle u \rangle$ is 1.506 and is located at 0.2 d_p from the wall while the smallest one is 0.579 at 0.5 d_p from the wall. At 0.8 d_p from the wall, the second largest value occurs which equals 1.323. It may be considered that $\Delta r = 0.025\ R = 0.268\ d_p$ is not small enough to provide local point velocity; therefore, a decreased width of $\Delta r = 0.01\ d_p$ is used for further investigation and is shown in Figure 3. At $\Delta r = 0.01\ d_p$, the maximum and minimum values of $\bar{u}(r)/\langle u \rangle$, evolve to be 1.663 and 0.508, respectively, and they are still at 0.2 d_p and 0.5 d_p from the wall. ($\langle u \rangle$ is average velocity.) However, the position of the second largest velocity moves from 0.8 d_p to 1.0 d_p from the wall, with its value between 1.323 and 1.354.

The prediction of velocity profiles by the Brinkman equation is shown in Figure 4 at the same time with those of the Ergun equation method developed in this article. As Figure 4 demonstrates, these two methods provide substantially different results. The Brinkman equation, which was originally restricted to low flow rates, has been extended to higher flow rates by incorporating the Ergun pressure loss relation (Vortmeyer and Schuster, 1983; Delmas and Froment, 1988; Tsotsas and Schlünder, 1988). Another extension of the Brinkman equation was made by Johnson and Kapner (1990) who considered the resistance offered by the packing to a flowing fluid as a function of the packing permeability. The packed bed was thus modeled as a fluid continuum which allows the packing resistance term to be added to the Navier-Stokes equation. The two forms of the Brinkman equation are identical in nature. Since the Ergun equation was derived based on the bed as a whole, its validity for local pressure drop estimation within the bed with porosity between 0.18 and 1.0 is naturally quite uncertain. Therefore, the prediction of the velocity profile by the Brinkman equation is rather questionable.

The effect of the wall to the hydraulic radius is shown in Figure 5. At $r = R$, which is actually at the wall, where there is only a wetted area and no flow space, the hydraulic radius ought to be zero. Although the free flow space per unit bed volume in the vicinity of the wall is high, the participation of the wall surface in forming the flow channel will make the hydraulic radius not too large.

The effect of particle diameters on velocity distributions is illustrated in Figure 6. It shows that the velocity profiles all exhibit reduced oscillating behaviors with a period of about one particle diameter. Specifically, the smaller the particle diameter, the higher the oscillation magnitude.

In order to study the wall effect in a fixed bed, the amount of volumetric flow within a distance of one particle diameter from the wall is of great importance. Figure 7 shows the amount of gas flow for particle sizes of 2 mm, 3.5 mm, and 7.5 mm in a 38 mm ID packed bed to be 21.0%, 35.1%, and 63.8% of the total, respectively.

The influence of the Reynolds number on the velocity distribution is also analyzed, as shown in Figure 8. This study covers a large interval from laminar flow ($R_{ep} = 11.8$) to suf-

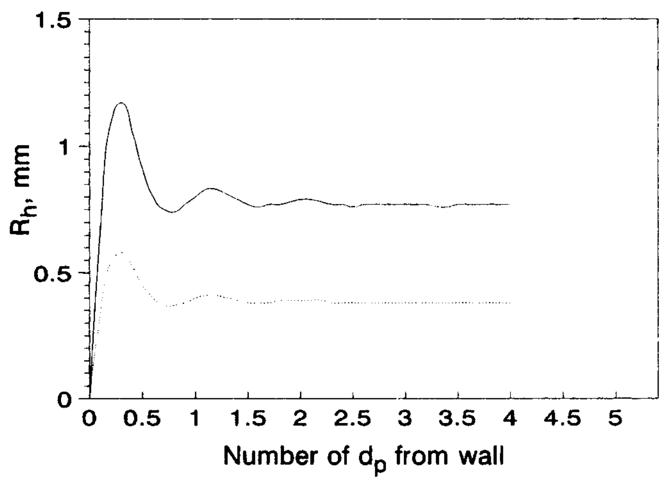

Figure 5. Hydraulic radius of region between wall and radial position *r*.

—— $d_t = 75.5$ mm; $d_p = 7.035$ mm; ···· $d_t = 38$ mm; $d_p = 3.5$ mm.

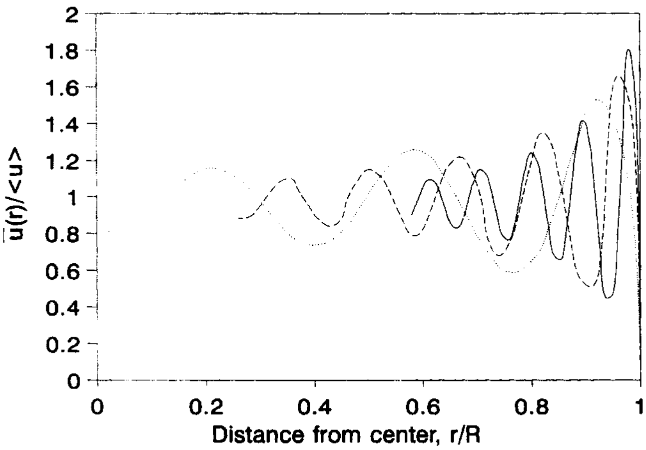

Figure 6. Velocity profiles at different pellet diameters.

—— $d_p = 2.0$ mm; ---- $d_p = 3.5$ mm; ···· $d_p = 7.5$ mm.

ficiently developed turbulent flow ($R_{ep} = 1{,}178.5$) (R_{ep} is the particle Reynolds number, $d_p u \rho/\mu$). It shows that the velocity nonuniformity is very pronounced for the laminar flow with the highest value of $\bar{u}(r)/\langle u \rangle$ approaching 2.5 and the lowest value almost zero. Such a result is reasonable since under laminar flow the average flow velocity is very low. The interstice is large enough for the fluid to pass. As the Reynolds number increases, the void space, which the fluid has been allowed to pass through freely under laminar flow, will become too congested. In this case, the flow channel will impose a large friction on the fluid, thus leading to a redistribution of the fluid over the entire cross section. It is interesting to note that in the turbulent regime, the velocity profiles do not differ by a large amount, and this is consistent with the above analysis.

Conclusions

Under low tube-to-particle diameter ratio, the wall effect of a fixed bed is important, therefore, it should be accounted for carefully. Based on the concept of effective diameter and

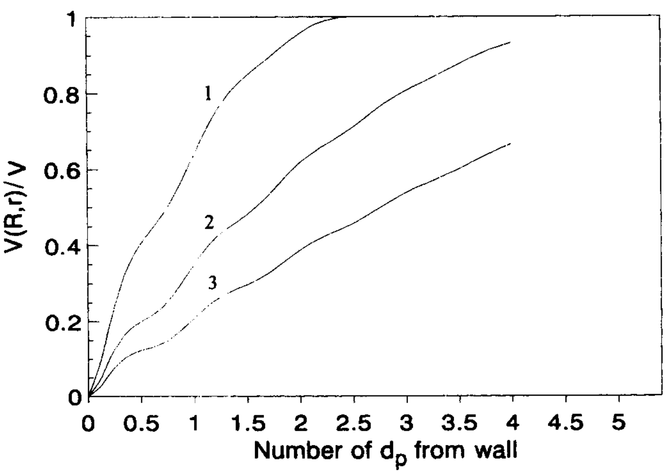

Figure 7. Volumetric flow ratio between wall and a certain radial position.

(1) $d_p = 7.5$ mm; (2) $d_p = 3.5$ mm; (3) $d_p = 2$ mm.

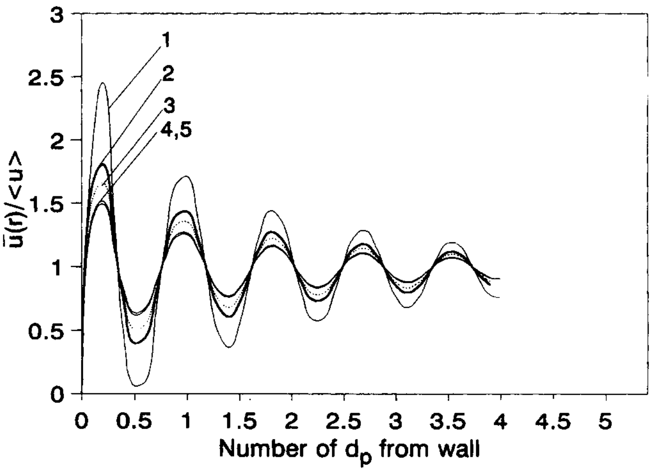

Figure 8. Velocity profiles at different Reynolds numbers.

(1) $R_{ep} = 1.8$; (2) $R_{ep} = 58.9$; (3) $R_{ep} = 117.9$; (4) $R_{ep} = 589.2$; (5) $R_{ep} = 1{,}178.5$.

by introducing it into the Ergun equation, a new method for estimation of velocity profiles is developed and leads to some conclusions:

(1) The predicted velocity profile is in good agreement both in magnitude and position with the optimal measurement of Stephenson and Stewart (1986). The maximum and minimum velocities are located at 0.2 d_p and 0.5 d_p from the wall, respectively. Specifically, at 1.0 d_p from the wall, where it was believed the position of the maximum velocity was, now turns out to be the second largest velocity.

(2) Pellet size for a bed of fixed diameter is an important factor influencing the velocity distribution. Also, the amount of bypass near the wall may be very remarkable when large pellets are used.

(3) The Reynolds number is also an important factor influencing radial velocity distribution, but is different from the packing size. The velocity distribution only varies with the Reynolds number when the flow velocity is low; however, in the turbulence regime, such as $R_{ep} > 200$, the velocity profiles remain unchanged.

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